2.29.1. Normal Form Problems

A. For each of the following truth tables, provide a sentence in **Conjunctive Normal Form** which takes that truth table.

Truth Table 1	Truth Table 2	Truth Table 3
1	0	1
1	1	0
0	1	1
0	1	0
1	0	1
1	1	1
1	1	1
1	0	1

B. Decide, for each of the following sentences, if it is in DNF, CNF, or neither.

1. $(\sim P \wedge Q) \vee (P \wedge \sim Q)$
2. $(P \vee \sim\sim Q) \wedge (\sim P \vee Q)$
3. $(Q \vee R \vee S) \wedge (\sim Q \vee \sim R \vee \sim S)$
4. $((Q \vee R) \wedge S) \vee ((\sim Q \vee \sim R) \wedge \sim S)$
5. $(P \vee (Q \wedge R)) \wedge (P \vee (\sim Q \vee \sim R))$
6. $(\sim(Q \vee R) \wedge S) \vee ((\sim Q \vee \sim R) \wedge \sim S)$

C. For any of the above sentences which are in DNF, restate the sentence in CNF. For any of the above sentences which are in CNF, restate the sentence in DNF.

D. The following two principles were noted in the text.

> **(i) The disjunction of some sentence with a contradiction is equivalent to that sentence alone.**
>
> **(ii) The conjunction of some sentence with a tautology is logically equivalent to that sentence alone.**

Using these two principles, along with Distribution [and DeMorgan’s Law], simplify each of the following sentences as much as possible.[1]

1. $(P \wedge (P \vee Q))$

2. $(P \vee (\sim P \wedge Q))$

[3. $(\sim P \wedge \sim(P \wedge Q))$]

E. For each of the following sentences in Disjunctive Normal Form, construct an equivalent sentence in Conjunctive Normal Form (without building truth tables). (Feel free to simplify disjunctions by eliminating contradictory cells, and conjunctions by eliminating tautologous cells.)

a. $(P \wedge \sim Q) \vee (\sim P \wedge Q)$
b. $(P \wedge \sim Q \wedge \sim R) \vee (\sim P \wedge \sim Q \wedge \sim R)$
c. $(\sim P \wedge Q \wedge \sim R \wedge S) \vee (\sim P \wedge \sim Q \wedge \sim R \wedge \sim S) \vee (\sim P \wedge Q \wedge \sim R \wedge \sim S)$

[1] All three sentences were first encountered in 2.15.1.

F. Convert each of the following sentences into both DNF and CNF, and decide on that basis whether the sentence is (i) a contradiction, (ii) a tautology, or (iii) neither.

Simplify disjunctions by eliminating contradictory cells, and conjunctions by eliminating tautologous cells. Also: feel free to remove inner parentheses from within multi-part disjunctions and conjunctions. For example, treat "$((P \wedge Q) \wedge R)$" as equivalent to "$(P \wedge Q \wedge R)$".

1. $((P \vee Q) \wedge {\sim}P) \wedge {\sim}Q)$
2. $((P \vee Q) \wedge P) \wedge {\sim}Q)$

G. Say that a DNF sentence is in **perfect DNF** if every cell of the sentence contains the same sentence letters, each appearing once. The following sentences, for example, are in perfect DNF, since each cell contains the same sentence letters – "P" and "Q" in the first sentence, "P," "Q," and "R" in the second.

$$(P \wedge Q) \vee ({\sim}P \wedge Q)$$
$$(P \wedge Q \wedge R) \vee ({\sim}P \wedge {\sim}Q \wedge {\sim}R)$$

1. State a general procedure for converting an 'imperfect' DNF into perfect DNF.

(Hint: while we've previously simplified conjunctions by deleting cells that are tautological, here would add a tautological disjunction and apply distribution.)

2. Put each of the following imperfect DNF sentences into perfect DNF.